# Mixed Coefficients from Different Bases for Power Amplifier Modeling

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Abstract—Finding an equilibrium between accuracy and reduced complexity in digital predistortion techniques is at the crux of modern linearization processes for power amplifiers. This work seeks to improve the trade-off between reducing the complexity and fidelity by analyzing a new approach of reduction algorithms: mixing two different bases before the reduction in order to expand the scope of data interpretation and selecting only the best coefficients from both bases and uniting them under a single, reduced basis.

Keywords—Radiofrequency, modeling, digital predistortion, Polar Volterra Series, Power Amplifiers

#### I. INTRODUCTION

Increasing relevancy of radiofrequency systems in the modern era has pushed for more efficient power amplifiers (PAs) [1]. However, the operation of PAs still requires highly linear outputs. Given the information that PAs have a trade-off between highly efficient zones and highly linear zones, other approaches are taken, in this case, linearization of the signal [2].

To linearize PA signal, many techniques are utilized. Amidst them, one of the commonly used ones is digital treatment. One of the digital treatment methods is to utilize equations that model the behavior of the PA to construct a basis that digitally represents the distortion of such PA, then utilizing the inverse of the behavior of distortion in order to linearize the output signal.

However, the number of parameters contained in this basis increases very rapidly with the polynomial order and memory length chosen, criteria that dictates the fidelity of the model, meaning that, for a reasonable fidelity, which usually results in a very large number of parameters. So large is this number that, for practical applications, it becomes unpractical, due to time needed to compute the output. A solution found for such problem is the reduction of parameters via reducing algorithms, such as incrementor and decrementor, ascending and descending algorithms [3]. These algorithms evaluate the contribution of each parameter to the fidelity of the modeling, and then select only the ones that contribute the most within a single basis of parameters.

The proposition of this work is to analyze what happens with the fidelity of the model when we mix two different bases and then reduce the coefficients utilizing one of the reducing algorithms. The study hereafter reports the modelling accuracy of the mixed basis of Polar Volterra Series (PVS) and Modified Angle Difference Series (MADS), as well as their respective standalone versions, both in its twodimensional (2D) form [4].

# II. PA MODELING AND 2D VERSIONS

In order to reduce the complexity of the systems, 2D restrictions were utilized, meaning that only were took into

consideration the unidimensional and bidimensional terms. Unidimensional terms are terms that only use the information of one instant of time, and bidimensional terms are those that only use information of two distinct instants of time. Applying such restrictions alterates the way the original bases, making them simpler and easier to analyze.

# A. Polar Volterra Series (PVS)

The Polar Volterra series is well-known within the digital predistortion community [5]. It has characteristics well suited to model memory-fading, nonlinear systems, such as PAs. However, in order to take into account many terms of memory, this system grows very fast in terms of numbers of coefficients, thus rendering the modeling unrealistically complex. Applying the 2D restriction contributes towards reducing significantly the number of coefficients, given by the constitutive equation :

$$\begin{split} y^{cal}(n) &= \sum_{p_1=1}^{P_1} \sum_{p_2=0}^{p_1-1} \sum_{m_1=0}^{M} \sum_{m_2=m_1+1}^{P_2} \sum_{p_3=1}^{P_3} \sum_{p_4=0}^{p_3-1} \times \\ &\sum_{l_1=0}^{L} \sum_{l_2=l_1+1}^{L} \sum_{p_5=1}^{p_3-1} \sum_{p_6=0}^{p_5-1} \sum_{l_3=0}^{L} \sum_{l_4=l_3+1}^{L} \times , (1) \\ &h_{p_1,p_2,p_3,p_4,p_5,p_6}(m_1,m_2,l_1,l_2,l_3,l_4) a^{(p_1-p_2)}(n-m_1) a^{p_2}(n-m_2) \\ &\left[ \left( e^{j\varphi(n-l_1)} \right)^{p_3-p_4} \left( e^{j\varphi(n-l_2)} \right)^{p_4} \right] \left[ \left( e^{-j\varphi(n-l_3)} \right)^{p_5-p_6} \left( e^{-j\varphi(n-l_4)} \right)^{p_6} \right] \end{split}$$

In (1) and (2), the first four sums regard amplitude information, and the remaining sums regard phase information.

A restriction in  $m_1, m_2, l_1, l_2, l_3, l_4$  is required: all parameters can only be referenced to two instants of time to force the 2D condition.  $m_1, m_2, l_1, l_2, l_3$ , and  $l_4$  can only assume two values  $n_1$  and  $n_2$ , those being any value smaller than M or L. For example: suppose  $m_1 = 1$ , then  $m_2, l_1$  and  $l_2$  can only be 1 or another memory number n. Suppose for sake of argument n=2. Then  $m_2, l_1, l_2, l_3$ , and  $l_4$  can only be 1 or 2. Looking at the 2D terms of this equation results in a reduced version of PVS, which can be evaluated and combined for further results.

# B. Modified Angle Difference Series (MADS)

Explored in the early work of [4], the MADS has similar properties to PVS, meaning it also has good for modeling memory fading, nonlinear systems. This, however, doesn't grow as fast in relation to the number of coefficients in proportion to the number of memory elements taking into consideration.

This should be a redeeming feature to the model, making it a better option than PVS. Unfortunately, its performance modeling PA behavior is inferior to PVS [4]. Meaning that, by itself, it is not a good option for modeling of PAs. Applying 2D restrictions also contributes towards the reduction of coefficients, given the constitutive equation:

$$y^{cal}(n) = \sum_{p_{1=1}}^{P_1} \sum_{p_{2=0}}^{p_1-1} \sum_{m_1=0}^{M} \sum_{m_2=m_1+1}^{M} \sum_{p_3=0}^{P_3} \sum_{l_1=0}^{L} \times \sum_{l_2=l_1+1}^{L} [h_{p_1,p_2,p_3}(m_1, m_2, l_1, l_2)] [a^{(p_1-p_2)}(n-m_1), (2) \times a^{p_2}(n-m_2)] [(e^{j\varphi(n-l_1)}e^{-j\varphi(n-l_2)})^{p_3}(e^{j\varphi(n)})]$$

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### C. Mixed Series (MIX)

This can be obtained by directly adding all coefficients of PVS and MADS. It is worth noting that due to the fact that PVS and MADS not containing the same coefficients, it is expected that when selecting the coefficients that contribute the most, better results might be found due to the combination of the bases.

#### III. ASCENDING APPROACH FOR COMPLEXITY REDUCTION

Many  $y^{cal}$  from sequential instants of time can be taken and ordered like a vector, the constitutive equations can be written like:

$$Y^{cal} = XH \tag{3}$$

where  $Y^{cal}$  is a N x 1 vector of the calculated outputs, using any given algorithm, X is a N x Q matrix whose elements are obtained from using a constitutive equation and taking parameters, H is a Q x 1 vector containing the coefficients, N is the number of samples being used to model the system and Q is the number of coefficients. In regard to what algorithm would be utilized for analysis, an ascending algorithm approach was chosen, because of its overall superb performance.

In this method, single coefficients are selected and the normalized mean square error (NMSE) contribution of that single coefficient is measured. Then they are ordered in ascending order, and the best ranked coefficient is selected. In the next step, single coefficients are selected and evaluated by the NMSE contribution of those coefficients in conjunction with the previously selected coefficient. Then the outcomes are ranked in ascending order, and the best ranked coefficient is selected. In the following steps, the processess of evaluating single coefficients with conjunction of the previously selected coefficients is repeated, until the desired number of coefficients is achieved.

NMSE can be calculated by the following equation:

$$NMSE = 10 \log_{10} \left\{ \frac{\sum_{n=1}^{N} |y_n^{des} - y_n^{cal}|^2}{\sum_{n=1}^{N} |y_n^{des}|^2} \right\}$$
, (4)

where  $y_n^{des}$  is the measured output at the time sample n and  $y_n^{cal}$  is the estimated output at the time sample calculated from any of the given bases.

# **IV. RESULTS**

In this section, the ascending algorithm is applied to the PVS, MADS and MIX for the modeling of the direct and inverse transfer characteristics from the PA under study. The inverse modeling has application in predistortion schemes.

Input and output data was collected, with a vector signal analyzer from Rohde & Schwarz, from a GaN class AB PA, with a center frequency of 900 MHz and subjected to a WCDMA 3.84 MHz envelope. The sampling frequency is 30.72 MHz.

In the PVS, MADS and MIX models, the truncation factors are set to M = 3, L = 2, P1 = 5 and P3 = 2. The ascending algorithm was executed in Matlab software using floating-point double-precision arithmetic. The input and output data was split in two: one part was utilized to set the coefficients and the other part was utilized to validate and test the bases.

Figures 1 and 2 show the calculated NMSE as a function of the number of coefficients Q, for the modeling of direct and inverse transfer characteristics, respectively. NMSE, being a measure of error, has to be a small number for the model to be accurate. That means that the lower a point in Fig. 1 and 2, the more accurate the given model is.



Fig. 1. NMSE versus the number of coefficients for direct modeling.



Fig. 2. NMSE versus the number of coefficients for inverse modeling.

Figures 1 and 2 show that MIX and PVS exchange best performance as the number of coefficients is changed,

meaning that for some number of coefficients, it is preferable to use MIX instead of PVS. This is a positive result, meaning that combining different bases can, sometimes, be utilized as a more effective alternative for digital modeling. Figures 3 and 4 show the relationship of the input amplitude and the output amplitude. The type of information shown in Figs. 3 to 6 are called amplitude modulation to amplitude modulation (AM-AM) characteristics. They are useful in evaluating how linear is the behavior of a given transfer characteristic in regards to the amplitude.



Fig. 3. AM-AM characteristics for direct modeling with 100 coefficients.



Fig. 4. AM-AM characteristics for inverse modeling with 100 coefficients.

Figure 5 and 6 also show the relationship of the input amplitude and the output amplitude in our new basis, but in different visualization.

Each sample is colored in relation to absolute value of the error of that given point and of its measured counterpart. The error parameter taken into account to color the results is the absolute value of the difference between the measured output, symbolized in the figures by the green markers, and the values calculated by the MIX basis. This approach was taken so that the position of the calculated value can be observed at the same time as the error of that position in relation to the expected output value. Bright blue meaning the measured and calculated were close and red meaning they were distant, meaning that the error was more significant, green markers simbolize the measured real value, for reference.



Fig. 5. AM-AM characteristics and error for direct modeling with 100 coefficients.



Fig. 6. AM-AM characteristics for direct modeling with 100 coefficients.

In Fig. 5 and 6, it can be observed that the number of red markers is sparse, in comparison to the number of blue markers, which means that, overall, the error of the proposed model, MIX, is pretty low, with exception of some outliers.

Figures 7 and 8 show the relationship of input amplitude and phase difference (output minus input). The type of information shown in Figs. 7 to 10 are called amplitude modulation to phase modulation (AM-PM) characteristics. They are useful in evaluating how linear is the behavior of a given transfer characteristic in regards to the phase.



Fig. 7. AM-PM characteristics for direct modeling with 100 coefficients.



Fig. 8. AM-PM characteristics for inverse modeling with 100 coefficients.



Fig. 9. AM-PM characteristics and error for direct modeling with 100 coefficients.



Fig. 10. AM-PM characteristics and error for inverse modeling with 100 coefficients.

Fig. 9 and 10 show the same information of Fig. 7 and 8 and the same approach to display information as the one used in Fig. 5 and 6. This is done here for the same reasons from

Fig. 5 and 6. That is, to observe different informations simutaneously in the same figure. It can be observed that the number of red markers is sparse and rare, meaning that the treatment of phase information on this model (MIX) is performing well.

It is clear, based on the observation of the results, that the models perform overall quite well, meaning that their performance in predicting single points is very good, with the exception of some outliers.

#### V. DISCUSSION

When using reduction algorithms, single basis has a certain performance, which is generated by the selection of the most contributing coefficients of that basis. Given the fact that these bases utilize a equation that delimits how coefficients interact with your data, selecting a finite amount N of best performing coefficients from that basis will render you the best N coefficients of that basis. Each basis you apply this algorithm will give you different N coefficients. Some bases will perform better with their N coefficients than others, but that does not necessarily mean that all N coefficients of the best performing basis are better than all N coefficients of the other bases. In essence, MIX only puts all coefficients of both bases in the same space and selects the N best performing coefficients within this collective basis. This treatment showed to be successful, for certain numbers of coefficients N, but not for all. What this implies is that, for some aplications, MIX basis might be a real choice. This choice must be made in function of which N the selected application wants to utilize. Referencing Fig. 1 and 2 or generating a similar figure might serve as guide for when to use such basis.

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